Enjoying, Believing, Achieving

Wychwood Church of England Primary School

Calculation Policy
February 2016

Wychwood C E Primary School

Calculation Policy

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as drawings, objects and empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental and written methods that they understand and can use correctly. When faced with a problem, children are able to make estimates, decide which method is most appropriate and have strategies to check its accuracy.

At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure and appropriate knowledge of vocabulary (everyday and mathematical) and number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave primary school they:

- have a good understanding of the everyday and mathematical language associated with the four operations to enable them to interpret problems in context and make appropriate strategy selections;
- have a secure knowledge of number facts and a good understanding of the nature of the four operations;
- are able to use their knowledge and understanding to carry out calculations mentally and to apply general strategies;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Practical, oral and mental work, in all Year Groups lays the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10, sums and differences of multiples of 10 and multiplication and division facts up to 10 × 10;
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number, partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine.
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers, and to calculate mentally with whole numbers and decimals.

Written methods of calculation

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient method for each operation with confidence and understanding. The challenge for teachers is determining when their children should move on to a refinement in the method and become more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental or written method based on the knowledge that they are in control of this choice as they are able to carry out a range of methods with confidence.

Notes:

- *Children need to recognise that 'ones' and 'units' are both commonly used to discuss the numbers 0 to 9 and therefore both are used in this policy.*

- *Children should understand that the = sign means 'the same as' i.e. the resulting value on either side of the sign is the same e.g. 3 x 4 = 48 ÷ 4*

Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10, (such as □ + 3 =10);
- add mentally a series of one-digit numbers, (such as 5 + 8 + 4);
- add multiples of 10 (such as 60 + 70) or of 100, (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways, and extend this strategy for larger and decimal numbers.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Throughout, written strategies should be supported by the development of everyday and mathematical vocabulary, the drawing of diagrams, handling of objects and the use of drama work in order to aid the understanding of contexts where addition methods are required.

Option 1: Recording and developing mental pictures

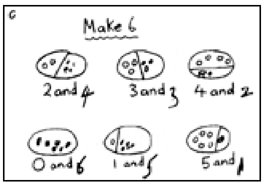
- Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.

Option 1

"One,

and one, two more

makes one, two three."



Option 2: Progression in the use of a number line

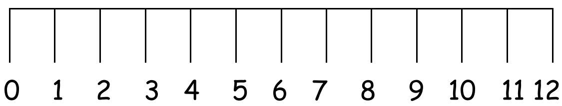
- To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines

Option 2

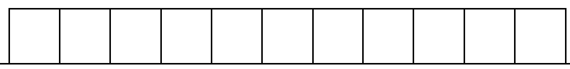
Number track

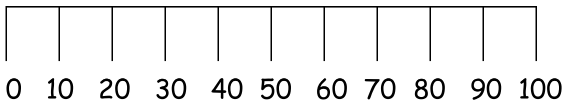
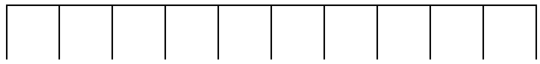
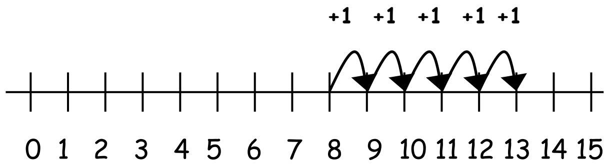
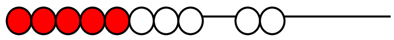
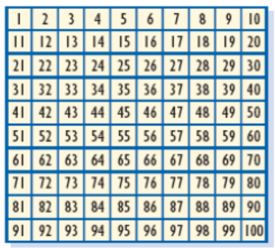
0	1	2	3	4	5	6	7	8	9	10	11	12

Number line, all numbers labelled



Number line, 5s and 10s labelled



	0 5 10 15 20 25 30 35 40 45 50 55 Number lines, 10s labelled  Number lines, marked but unlabelled  Empty number line
The labelled number line • Children begin to use numbered lines to support their calculations counting on in ones. • They select the biggest number first i.e. 8 and count on the smaller number in ones.	8 + 5 = 13 
Additional 'number lines; - The bead string and hundred square • Along with the number line, bead strings can be used to illustrate addition. Eight beads are counted out, then the two beads. Children count on from eight as they add the two beads e.g. starting at 8 they count 9 then 10 as they move the beads. • Eight beads are counted out, then the five. Children count on from eight as they add the five e.g. starting at 8 they count 9, 10, 11, 12, 13. • A hundred square is an efficient visual resource to support adding on in ones and tens.	8 + 2 = 10  8 + 5 = 13  

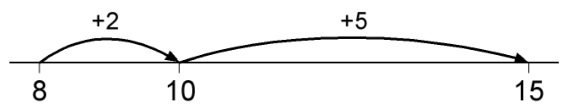
The empty number line

- The mental methods that lead to column addition generally involve partitioning.
- Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.
- The use of a number line can be extended successfully to deal with the addition of decimal numbers.

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

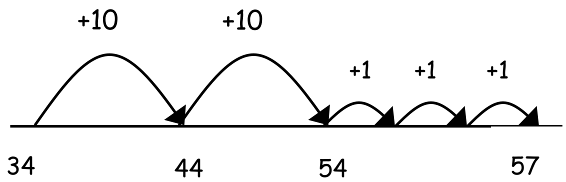
8 + 7 = 15

Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.



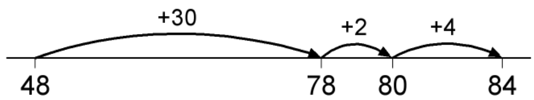
First counting on in tens and ones.

34 + 23 = 57



Then counting on in multiples of 10.

48 + 36 = 84



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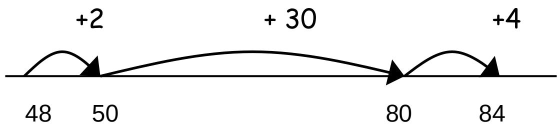

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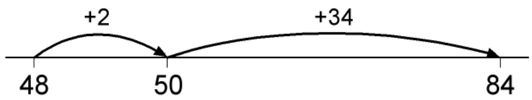
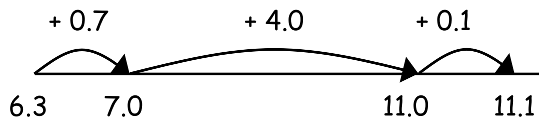


6.3 + 4.8 = 11.1



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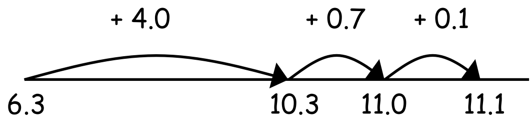
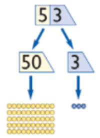


Option 3: Partitioning

- The next option is to record mental methods using partitioning into tens and ones separately.



Partitioning into tens and ones.

- Add the tens and then the ones to form partial sums and then add these partial sums.

- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

- This method can be extended for TU + HTU and HTU + HTU and beyond; as well as catering for the addition of decimal numbers.

Option 3

Record steps in addition using partitioning:

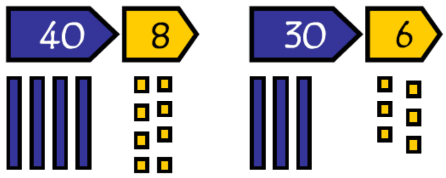
47 + 76
40 + 70 = 110
7 + 6 = 13
110 + 13 = 123

or 47 + 76
7 + 6 = 13
40 + 70 = 110
110 + 13 = 123

or 47 + 76
47 + 70 + 6 = 117
117 + 6 = 123

Such calculations can be supported by the use of place value cards and Dienes apparatus e.g.

48 + 36



40 + 30 = 70
8 + 6 = 14

70 + 14 = 84

Partitioned numbers are then written under one another, for example :

	T U
48	40 + 8
+ 36	+ 30 + 6
	70 + 14 = 84

155 + 267

	H T U
155	100 + 50 + 5
+ 267	+ 200 + 60 + 7
	300 + 110 + 12 = 422

4.6 + 12.7

	T U th
4.6	4 + 0.6
+ 12.7	+ 10 + 2 + 0.7
	10 + 6 + 1.3 = 17.3

Option 4: Expanded method in columns	Option 4
• This layout shows the addition of the tens to the tens and the ones to the ones separately. To find the partial sums initially the tens, not the ones, are added first, following mental methods. The total of the partial sums can be found by adding them together. • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • As children gain confidence, ask them to start by adding the ones first every time. Explain that adding the units first leads the strategy towards the next option, the compact method. • This expanded method leads children to the more compact method in such a way that enables them to understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	Write the numbers in columns. Adding the tens first: 47 + 76 110 13 123 Place value headings may be written above the columns to aid the understanding of the value of the numbers. Adding the ones first: T U 47 + 76 13 110 123 Discuss how adding the ones first will give the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.

Option 5: Compact column method

- In this method, recording is reduced further. Carried digits are recorded above the line, using the words 'carry ten' or 'carry one hundred' etc, according to the value of the digit.

- Later the method is extended when adding more complex combinations such as four and five -digit numbers, and problems involving several numbers of different sizes including decimals.

Option 5

$$\begin{array}{r} 324 \\ +\ 439 \\ {\scriptstyle 1\ } \\ \hline 763 \\ \hline \end{array} \qquad \begin{array}{r} 576 \\ +\ 648 \\ {\scriptstyle 1\ 1\ } \\ \hline 1224 \\ \hline \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

$$\begin{array}{r} 3674 \\ 2507 \\ +\ 4175 \\ {\scriptstyle 1\ 1\ 1\ } \\ \hline 10356 \\ \hline \end{array} \qquad \begin{array}{r} 45873 \\ +\ 36758 \\ {\scriptstyle 1\ 1\ 1\ 1\ } \\ \hline 82631 \\ \hline \end{array}$$

$$\begin{array}{r} 36.8 \\ +\ 24.7 \\ {\scriptstyle 1\ 1\ } \\ \hline 61.5 \\ \hline \end{array} \qquad \begin{array}{r} 6.253 \\ +\ 14.562 \\ {\scriptstyle 1\quad 1\ } \\ \hline 20.815 \\ \hline \end{array}$$

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 and 100 (such as 160 – 70, 1600 - 700) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14), extending this strategy for larger and decimal numbers.

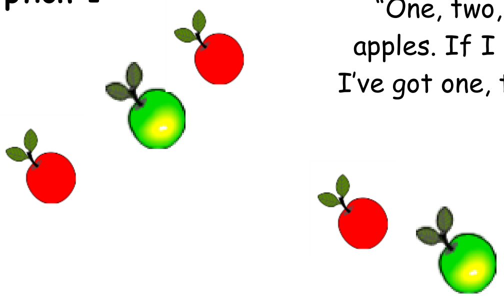
It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Throughout, written strategies should be supported by the development of everyday and mathematical vocabulary, the drawing of diagrams, handling of objects and the use of drama work in order to aid the understanding of contexts where subtraction methods are required.

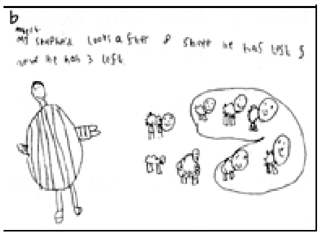
Option 1: Recording and developing mental pictures

- Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures.

Option 1



"One, two, three apples. If I eat one, I've got one, two left."



Option 2: Progression in the use of a number line

- Finding out how many items are left after some have been 'taken away' is initially supported with a number track followed by labelled, unlabelled and finally empty number lines, as with addition.

Option 2

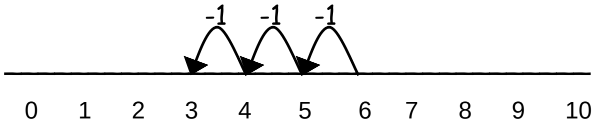
Number track

0	1	2	3	4	5	6	7	8	9	10	11	12

The labelled number line

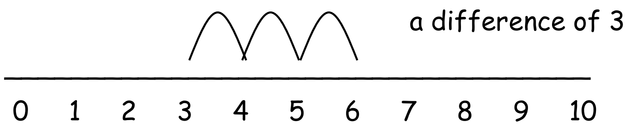
- The labelled number line is used to support calculations where the result is less objects (i.e. taking away) by counting back.

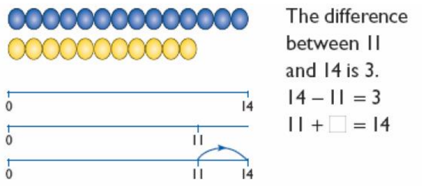
6 - 3 = 3



Difference between

- The number line should also be used to make comparisons between numbers, to show that 6 – 3 means the '**difference in value** between 6 and 3' or the 'difference between 3 and 6' and how many jumps they are apart.

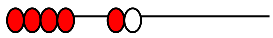




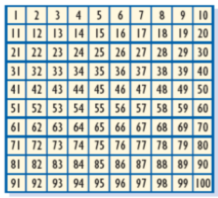
Additional 'number lines' - The bead string and hundred square

- Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.
- A hundred square is an efficient visual resource to support counting on and back in ones and tens.

6 - 2 = 4



13 – 5 = 8



The empty number line

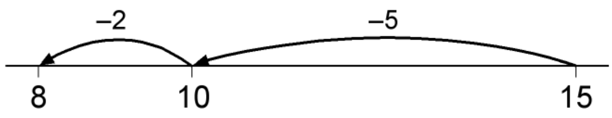
Finding an answer by COUNTING BACK

- **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers e.g. the 7 is partitioned into 5 and 2 to enable counting back to 10.
- The empty number line helps to record or explain the steps in mental subtraction.
- A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.

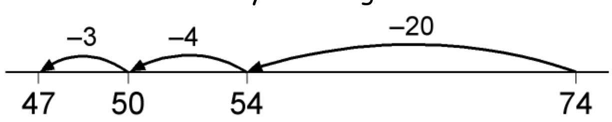
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7 = 8

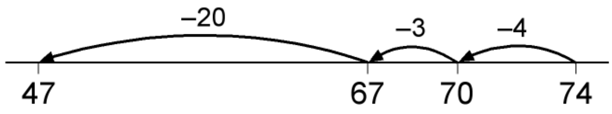
The seven is partitioned into 5 (to allow count back to 10) and two.



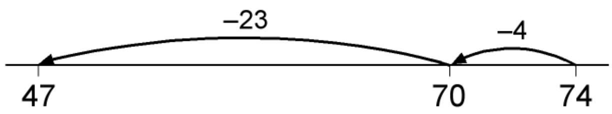
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined:



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Using an empty number line

Finding an answer by COUNTING ON

- The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm. How much longer is Alex's scarf?

- **With practice, children will need to record less information and *they will decide whether to count on or back.***

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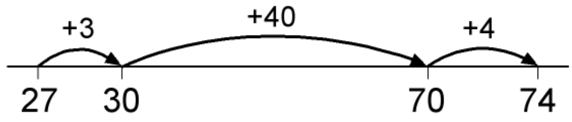
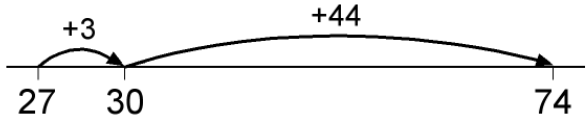
74 – 27 =



The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3.

or:



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- With three-digit numbers, the number of steps can again be varied, enabling children to work out answers to calculations such as 326 – 178 first in small steps and then in fewer steps by using their knowledge of complements to 100.
- The most compact form of recording becomes reasonably efficient.
- Addition of the 'jumps' begins with the most significant figures (the largest place value).

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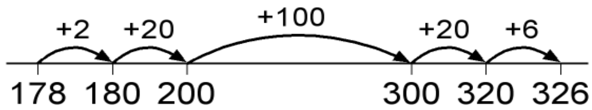
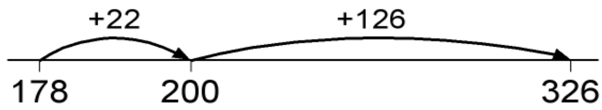
326 – 178 =



= 100 + 40 +8 = 148

or:



= 126 + 22 + 148

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- The method can successfully be used with decimal numbers.

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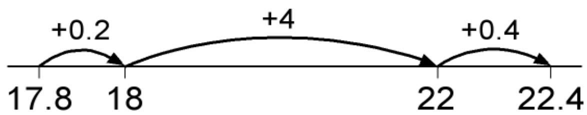
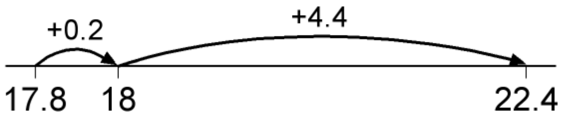
22.4 – 17.8 =



= 4 + 0.6 = 4.6

or:



= 4.4 + 0.2 = 4.6

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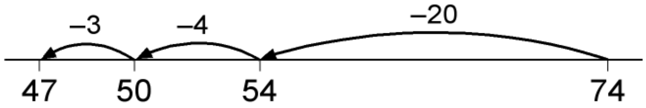
Option 3: Partitioning	**Option 3**
• Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn. A context might be: Bill has £74. A pair of football boots cost £27. How much will he have left?	Subtraction can be recorded using partitioning: 74 - 27 74 - 20 = 54 54 - 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line. 

Option 4: Expanded layout, leading to column method (Decomposition)

- Decomposition requires the partition of numbers and in some cases adjustment. Adjustment involves exchanging quantities of one place value for a lower place value e.g. exchanging a ten for 10 ones or a hundred for 10 tens.
- Partitioned numbers in hundreds, tens and ones are written one under the other mirroring the column method, where ones are placed under ones and tens under tens etc.
- This does not link directly to mental methods of counting on or back but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method in such a way that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Expanded method

Example (a): 563 – 241. The numbers are partitioned but no adjustment is required.

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array}$$

The larger number, which is being subtracted from, is placed above the smaller number. There is no exchange required as each partitioned component of the larger number is greater than that of the smaller number.

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying '**sixty** take away **forty**', not 'six take away four'. Similarly with the hundreds, say '**five hundred** take away **two hundred**'.

Place value headings may be written above the columns to aid the understanding of the value of the numbers.

Example (b): 563 – 246

There are insufficient units therefore adjustment is required from the tens to the units.

			50		13		
	500	+	~~60~~	+	~~3~~		
-	200	+	40	+	6		
	300	+	10	+	7	= 317	

A ten is exchanged for ten units so instead of 60 + 3 we have 50 + 13; the value is equivalent.

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, how there is a "snag" with the units. We have only 3 units and we need to take away 6. In order to do this we need to gain more units. To release the units 60 + 3 can be partitioned into 50 + 13. The subtraction of the units becomes '13 minus 6'.

Example (c): 563 – 271

There are insufficient tens therefore adjustment is required from the hundreds to the tens,

	400		160			
	~~500~~	+	~~60~~	+	3	
-	200	+	70	+	1	
	200		90		2	= 292

A hundred is exchanged for ten tens so instead of 500 + 60 we have 400 + 160; the value is equivalent.

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, how there is a "snag" with the tens. We have a value of 60 from which we need to take away 70. We need to exchange a hundred to release the needed tens. 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70.

Example (d): 563 – 278

There are insufficient tens and ones therefore adjustment from the hundreds to the tens and the tens to the ones is required.

	400		150			
			~~50~~		13	
-	~~500~~	+	~~60~~	+	~~3~~	
	200	+	70	+	8	
	200	+	80	+	5	= 285

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits being subtracting from. First by using exchange, 60 + 3 is partitioned into 50 + 13 to provide more units. The units can be subtracted (13 – 8) however in the tens column we cannot take 70 from 50. A hundred also needs to be exchanged to release 10 more tens; hence 500 + 50 is partitioned into 400 + 150, to help subtraction to take place.

Example (e): 503 – 278 Dealing with zeros when adjusting.

```
            90      13
   400     1̶0̶0̶
   5̶0̶0̶  +  0̶   +   3
-  200  +  70  +   8
   200  +  20  +   5    = 225
```

Here '0' acts as a place holder for the tens in the number being subtracted from. In order to supply units, the adjustment has to be done in two phases. First, by exchanging a hundred for 10 tens, the 500 + 0 becomes partitioned into 400 + 100. Having created some tens, one can be exchanged for units resulting in the 100 + 3 being partitioned into 90 + 13.

Please note that, when calculating with numbers close to a multiple of 100 or 1000, it would probably be more efficient to use a mental method or a number line.

Option 5: Compact method for three-digit numbers

NB: It is initially useful for the expanded method to be shown alongside compact method to aid children's understanding of the link.

Example (a): 563 – 241, no adjustment or decomposition needed

```
  500 + 60 + 3        563
- 200 + 40 + 1       -241
  300 + 20 + 2        322
```

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example (b): 563 – 246, adjustment from the tens to the units

```
            50     13                5 1
   500  +  6̶0̶  +  3̶                 56̶3
-  200  +  40  +  6             -  246
   300  +  10  +  7   = 317        317
```

Ensure that children can explain the compact method, referring to the real value of the digits. They need to understand that, by using exchanging, they are repartitioning the 60 + 3 as 50 + 13.

Example (c): 563 – 271, adjustment from the hundreds to the tens, or partitioning the hundreds

```
   400     160                       4 1
   5̶0̶0̶  +  6̶0̶  +   3                 5̶63
-  200  +  70  +   1            -   271
   200     90      2   = 292        292
```

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160 by using exchanging a hundred for 10 tens. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Ensure that children can explain confidently how the numbers are repartitioned and why.

Example (d): The inclusion of zeros can make the adjustment more complex. Here, there are insufficient tens, however there also no hundreds. A thousand is exchanged for 900 and 100. The 900 remains in the hundreds column and the 100 is added to the 50 in the tens column so that 60 can be subtracted.

3000   900   150
~~4000~~ + ~~0~~ + ~~50~~ + 6
\- 2000 + 700 + 60 + 4
1000 + 200 + 90 + 2

3 9 1
~~4~~ 0 5 6
\- 2 7 6 0
1 2 9 2

Example (e): This method can be extended to involve numbers of different sizes including decimals.

4 11   1
~~5~~ ~~2~~ . 5 6
\- 1 5 . 7
3 6 . 8 6

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 12 × 12;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5, 700 × 50 or 0.7 x 5 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using efficient addition methods (see above).

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Throughout, written strategies should be supported by the development of everyday and mathematical vocabulary, the drawing of diagrams, handling of objects and the use of drama work in order to aid the understanding of contexts where multiplication methods are required.

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Option 1: Recording and developing mental images

- Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s.
- They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.
- They develop ways of recording calculations using pictures, etc.
- They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning answering questions such as; 'How many eggs would we need to fill the egg box? How do you know?'
- Children will use repeated addition to carry out multiplication supported by the use of counters/cubes.

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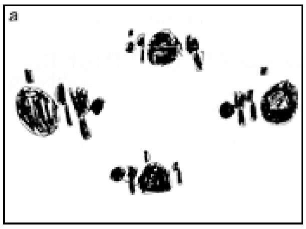
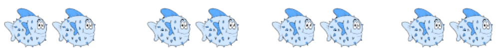
Option 1





2 + 2 + 2 + 2

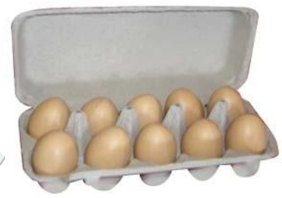
2 + 2 + 2 + 2 = 8

4 lots of 2 is 8

2 times 4 is 8

4 multiplied by 2 is 8

2 groups of 4 is 8



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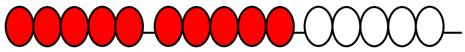
Option 2: The bead string, number line and hundred square

- Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line.
- On a bead string, children count out three lots of 5 then count the beads altogether.
- On a number line. Children count on in groups of 5.
- These models illustrate how multiplication relates to repeated addition.
- Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.

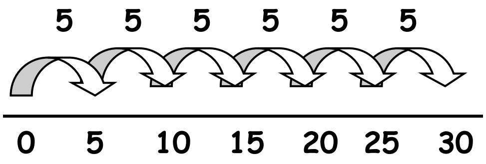
Option 2

3 lots of 5

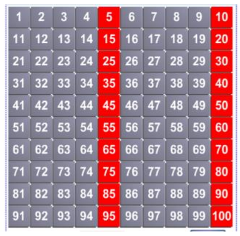
5 + 5 + 5 = 15



6 X 5 =



Multiples of 2



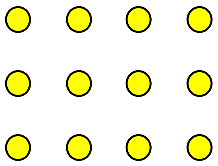
Multiples of 5

Option 3: Arrays

Successful written methods depend on visualising multiplication as a rectangular array. The array helps children to understand why 3 X 4 = 4 X 3

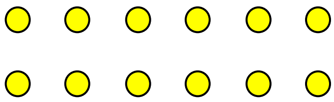
3 lots of 4

3 X 4



4 lots of 3

4 x 3

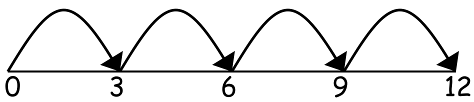


2 lots of 6

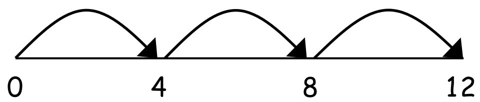
2 X 6

6 lots of 2

6 X 2



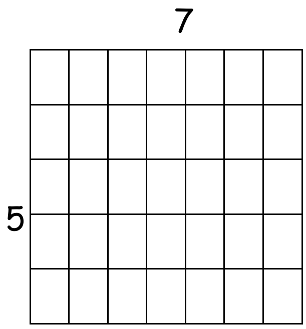
4 lots of 3 = 12

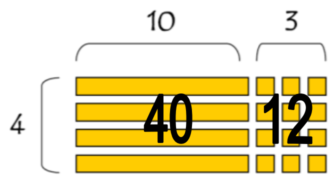


3 lots of 4 =12

- The rectangular array gives a good visual model for multiplication. The number of blocks can be found by repeated addition (in this case 7+7+7+7+7 = 35 or 5+5+5+5+5+5+5 = 35)
- Children should then commit 7 X 5 to memory and know that it is the same as 5 X 7.

7 X 5 = 35 or 5 x 7 = 35



• **Note:** Area models like this discourage the use of repeated addition. The focus is purely on the multiplication facts and therefore should be avoided.	7 X 5 = 35 or 5 x 7 = 35 7 5 | 35
Option 4: The Grid Method • The use of arrays leads on the grid method of written multiplication beginning with TU x U. • The TU number is partitioned e.g. 13 becomes 10 and 3 and each part of the number is them multiplied by 4. • Place value apparatus such as Dienes can be used to model the strategy. • Place value headings may written above to aid the understanding of the value of the numbers	**Option 4** 4 x 13  | | 10 | 3 | | 4 | 40 | 12 | 40 + 10 + 2 = 52
Related calculations and estimates • Children need to be able to identify and use related calculations and place value effectively to utilise this method. • For 38 X 7 they must be able to calculate 30 X 7. They need to recognise the 'root' calculation 3 x 7 = 21 and understand that as 30 is ten times greater than 3 the product will also be ten times greater. 30 x 7 = 210 • Before carrying out calculations children are encouraged to estimate their answer using rounding. In the case of a 2 digit number to the nearest 10 and a 3 digit number usually to the nearest 100. They compare their answer with the estimate to check for reasonableness.	Estimate: 38 X 7 is approximately 40 X 7 = 280 | | 30 | 8 | | | 7 | 210 | 56 | = 266 | 210 + 50 + 6 = 266 Using the estimate to check and answer: As the rounded number, 40, is slightly greater than 38 the answer to 38 x 7 would be expected to be slightly less than 280.

Two-digit by two-digit products using the grid method (TU x TU)

- Children first make an estimate by rounding each number to the nearest 10.
- Having calculated the sections of the grid, children will decide whether to add the rows or columns first as they become more confident with recognising efficient calculations.
- They will choose jottings, informal or formal written methods depending upon which is most appropriate.
- Comparison with the estimate here shows quite a large difference however children should take into account rounding down from 14 to 10 is a considerable difference. As children gain experience and confidence they will be able to employ rounding to the nearest 5 to gain a more accurate estimate e.g.

  38 x 14 becomes 40 x 15 = 600.

38 × 14

Estimate: 38 x 14 is approximately 40 X 10 = 400.

	30	8	
10	300	80	= 380
4	120	32	= 152

Adding the rows

	30	8
10	300	80
4	120	32
	420	112

Adding the columns

A possible method for finding the final answer might be partitioning carried out as a written or mental process e.g. using the row totals

380 + 152 =

300 + 100 = 400

80 + 50 = 130

0 + 2 = 2

400 + 100 + 30 + 2 = 532

Three-digit by two-digit products using the grid method (HTU × TU)

- Extend to HTU × TU asking children to estimate first and check their answer against the estimate.
- Children will choose an appropriate method for adding the column or row products according to their experiences and confidence with strategies.
- Ensure that children can explain why this method works and where the numbers and the grid come from.

138 x 24

Estimate: 138 X 24 = is approximately 140 X 25 = 3500

	100	30	8
20	2000	600	160
4	400	120	32
	2400	720	192

A child may choose to add these numbers using a column addition method either extended or compact as show below:

```
  2400
   720
+  192
  11
  ----
  3312
```

Or partitioning:

2000 + 1200 + 110 + 2 = 3312

Using the grid method for decimals

- The grid method works just as effectively with decimal numbers as long as the children can apply their knowledge of multiplication facts and place value.

38.5 X 24

Estimate: 38.5 X 24 is approximately 40 X 25 = 1000

	30	8	0.5	
20	600	160	10	770
4	120	32	2	154

Children will choose a method to add the column or row totals such as partitioning:

700 + 100 + 70 + 50 + 4 =

800 + 120 + 4 = 924

Option 5: Expanded short or compact multiplication • The first step is to present the numbers in a column format and write down each step of the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.	**Option 5** 38 X 7 is approximately 40 X 7 = 280 30 + 8 x 7 210 30 X 7 56 8 X 7 266 Multiplying the tens first follows on from the grid method. However introducing multiplying the units first enables children to move towards the compact method e.g. 30 + 8 X 7 56 7 x 8 210 7 x 30 266
Short or compact multiplication (TU x U) • The recording is reduced further, with the carried digits recorded above the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format or the grid method.	38 X 7 is approximately 40 X 7 = 280 38 X 7 5 266 The step here involves adding 210 and 50 mentally with only the '5' of the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a three-digit number mentally before they reach this stage.

TU x TU • Multiplying two-digit by two-digit numbers initially includes each step of the working to emphasise the link with the grid method starting with the most significant figures (20 × 50).	56 × 27 is approximately 60 × 30 = 1800. 56 x 27 1000 20 x 50 = 1000 120 20 x 6 = 120 350 7 x 50 = 350 42 7 x 6 = 42 1 1512 However introducing multiplying the units/ones first enables children to move towards the compact method e.g. 56 x 27 42 7 x 6 = 42 350 7 x 50 = 350 120 20 x 6 = 120 1000 20 x 50 = 1000 1 1512
Short or compact multiplication **TU x TU** • The first step, multiplying 56 by 7, involves mentally adding 42 to 350 without any recording. • The next step is to multiply 56 by 20 again mentally adding 120 to 1000 before recording. • The final phase shown here uses compact addition.	56 X 27 is aprroximately 60 x 30 = 1800 56 x 27 392 1120 1 1512

Three-digits multiplied by 1-digit and 2 digit numbers (HTU x U and HTU x TU) Although an expanded method could be used it becomes cumbersome. Depending upon the confidence of the child, the carried figures may be carried above the line or not when multiplying by 1-digit. When multiplying by 2-digits children need the confidence to add the carried figures mentally. Children who are less confident with the compact method would be advised to use their grid multiplication skills.	673 x 7 Estimate: 700 x 7 = 4900 673 x 7 5 2 4711 673 x 7 4711 286 x 27 Estimate: 300 x 25 = 7500 286 x 27 2002 5720 7722
Four-digits multiplied by 1-digit and 2-digit numbers and decimal numbers Depending upon the confidence of the child, the carried figures may be placed above the line or not. This methods can be similarly used for the multiplication of decimals.	6451 x 8 Estimate: 6000 x 8 = 48 000 6451 x 8 51608 58.73 x 7 411.11 2373 x 32 Estimate: 2000 x 30 = 60 000 A closer estimate would be: 2300 x 30 = 69 000 2373 x 32 4746 71190 1 75936

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

To divide successfully in their heads, children need to be able to:

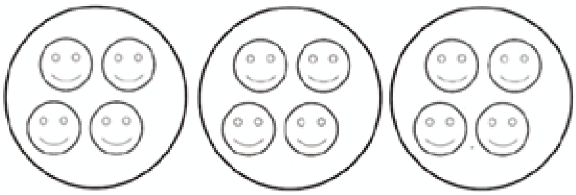
- understand and use the vocabulary of division (grouping and sharing);
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 12 × 12 recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successfully, children also need to be able to:

- understand division as the practical activities of both grouping and sharing;
- understand division can be carried out as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g. 120 - 80, 320 – 90.

Throughout, written strategies should be supported by the development of everyday and mathematical vocabulary, the drawing of diagrams, handling of objects and the use of drama work in order to aid the understanding of contexts when division methods are required.

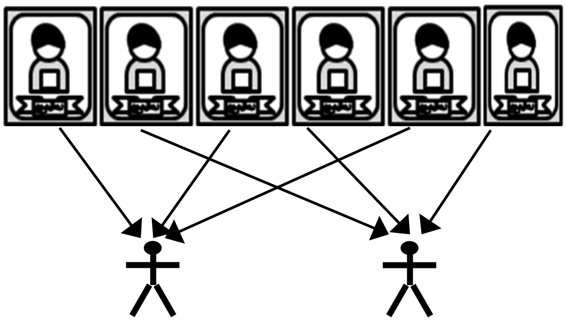
Option 1: Recording and developing mental images	**Option 1**
• Children are encouraged, through practical experiences, to develop physical and mental images. • They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.	

Sharing and Grouping

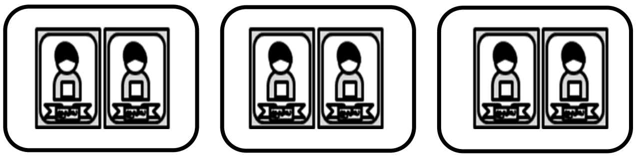
- They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out.
- Children should find the answer by counting how many cards **1 person** has got.

- They solve grouping problems by creating groups of the given number.
- Children should find the answer by counting out the stickers and finding out how many **groups of 2** there are.
- They will begin to use their own jottings to record division

6 football stickers are **shared** between 2 people, how many do they each get?



There are 6 football stickers, how many people can have 2 stickers each?



Option 2: Bead strings, number lines simple multiples

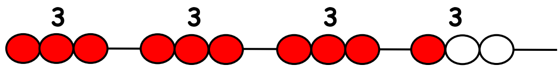
- Using a bead string, children can represent division problems.
- They count on in equal steps based on adding multiples up to the number to be divided.
- When packing cakes into boxes of three they count in threes – **grouping**.

- If the problem requires 12 cakes to be **shared** between 3 people, the multiple of three is obtained each time all three people have received a cake.

Option 2

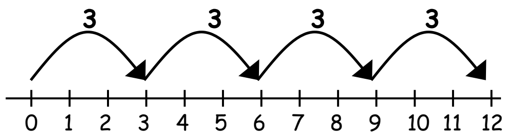
12 cakes are packed 3 in each box. How many boxes are needed?

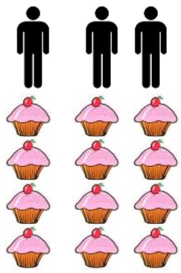
12 ÷ 3 = 4 boxes



Counting on using a labelled number line.

12 ÷ 3 = 4





3 cakes once

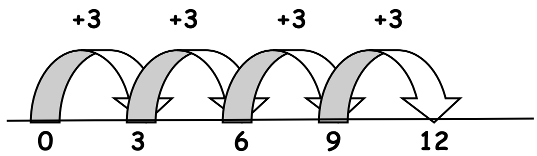
3 cakes twice

3 cakes three times

3 cakes four times

Counting on using a blank number line.

12 ÷ 3 = 4



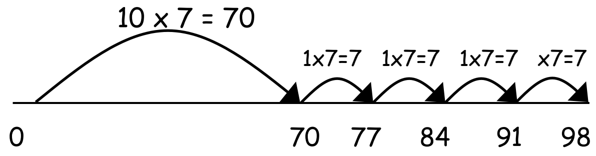
Each person has 4 cakes.

Option 3: Counting on by chunking

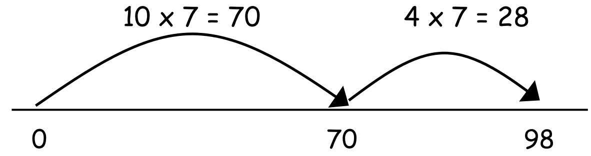
- This method is based on employing multiples of the divisor, or 'chunks'.
- Initially children begin to look for a 'chunk' of 10 of the multiple.
- The 'remainders' (mathematical language) can be discussed as 'left overs' (everyday language) as this leads to a more visual image when exploring what to do with anything that has been left after sharing or grouping. Some 'remainders' can be 'cut up' and shared leading to fractional and decimal parts others can not 'cut up' and discussion is needed to decide what to do with these according to the context of the situation.

Option 3

100 ÷ 7 =



or 100 ÷ 7 =

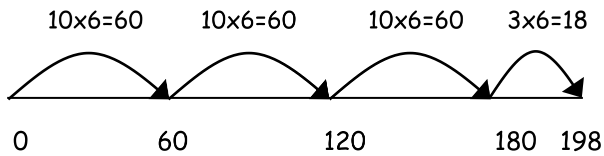


Answer: 14 remainder 2

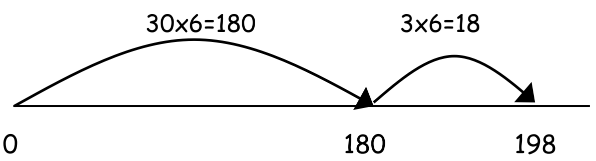
When discussing division, ask: 'How many sevens are in 100?' as well as 'What is 100 divided by 7?' to help children relate division to its inverse multiplication.

- Initially children employ several 'chunks' of 10 of the multiple.
- With practice they should look for the biggest multiples of 10 of the divisor that they can find to subtract e.g. 30 x 6.
- Children need to recognise that chunking is inefficient if too many steps have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.
- This strategy can be extended for larger numbers where 'chunks' of a 100 of the multiple are employed.

200 ÷ 6



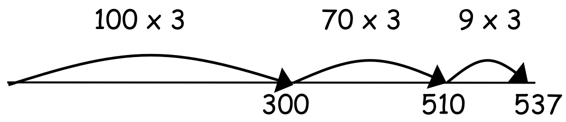
200 ÷ 6



Answer: 33 remainder 2

When discussing division ask: 'How many sixes in 200?' as well as 'What is 200 divided by 6?'

537 ÷ 3



Answer: 179

Option 4: 'Expanded' method for TU ÷ U recorded in columns • This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. • As you record the division, ask: 'How many sixes in 90?' or 'What is 90 divided by 6?' • This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps as illustrated in Option 3, when using a number line.	**Option 4** 96 ÷ 6 = To find 96 ÷ 6, we start by multiplying 6 by 10, to find that 6 × 10 = 60 and 6 × 20 = 120. The multiples of 60 and 120 trap the number 96. This tells us that the answer to 96 ÷ 6 is between 60 and 120. Start the division by first subtracting 60 leaving 36, and then subtracting the largest possible multiple of 6, which is 36, leaving no remainder. 96 - 60 10 X 6 36 - 36 6 X 6 0 Answer: 16 **Estimation** More than 10x6=60 but less than 20x6=120
Option 4a: Short method for TU ÷ U Alongside the expanded method children will begin to explore short division methods and learn how the two methods are related. Children use their knowledge of place value and think about how many multiples of 10 of 7 there are in 90. Their estimations support this work. They can only make 10 lots of 7. The 10 is placed in the tens column above the 90. The remaining 20 is carried to the units/ones column. They then ask how many 7s are in 28 and place the 4 in the units column.	98 ÷ 7 becomes **1 4** **7** | **9** ^{2}**8** Answer: 14 **Estimation** More than 10x7=70 but less than 20x7=140

Option 5: 'Expanded' method for HTU ÷ U • Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same. • The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend. • Estimating has two purposes when doing a division: – to help to choose a starting point for the division; – to check the answer after the calculation.	196 ÷ 6 = To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Initially children will subtract chunks about which they are totally confident. Here a series of chunks (6 X 10) are subtracted to reach 16 then 6 X 2 until no more whole sixes are left, leaving a remainder of 4. 196 - 60 10X6 136 - 60 10X6 76 - 60 10X6 16 - 12 2X6 4 32 Answer: 32 r 4 **Estimation** More than 30X6= 180 but less than 40X6=240
• Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.	Here the child has been confident to use the largest possible multiple of 10 as the initial multiplier. Start the division by first subtracting 180 (6 X 30), leaving 16 and then subtracting the largest possible multiple of 6 (which is 12) leaving 4. 196 - 180 30X6 16 - 12 2X6 4 32 Answer: 32 r 4 **Estimation** More than 30X6= 180 but less than 40X6=240 The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Option 5a: Short method for HTU ÷ U and Th H T and U • The next step is to tackle HTU ÷ U using short division methods. • When children have become confident in their understanding of place value they change their strategy and will say 5 into 4 won't go; how many 5s in 43, 8 and carry the remaining 3 to the units column; finally, they ask how many 5s in 32, 6 and a remainder of 2. • Children extend this strategy to the division of larger numbers and decimals.	432 ÷ 5 becomes **8 6 r 2** **5 │ 4 3 ³2** Answer: 86 remainder 2 **Estimation** More than 5 x 80 = 400 but l 1256 ÷ 6 = 2 0 9 r2 6) 1 2 5 ⁵6 Answer : 209 r2 34.73 ÷ 7 = 4 . 9 6 r 0.01 7) 3 4 . ⁶7 ⁴3 Answer: 4.96 r 0.01
Option 6: Long division • The next step is to tackle HTU ÷ TU • Children will as how many 24s can be made from 56, and place the 2 in the tens column as we are actually asking how many 24s in 560. The remaining 8 (80) is carried to the unit column. Children then ask how many 24s in 80, 3 and 8 remaining. • Initially they find writing the number sentences alongside their working.	560 ÷ 24 = How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 24) 560 - 480 24 x 20 80 - 72 24 x 3 8 Answer: 23 r 8 **Estimation** More than 24 x 20 = 480 but less than 24 x 30 =720 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. 23 r8 24) 560 - 480 80 - 72 8 Answer: 23 r 8

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Option 6a: Long division with larger numbers and decimal numbers

- The above methods are then extended to deal with 4 and 5-digit numbers and decimal numbers.

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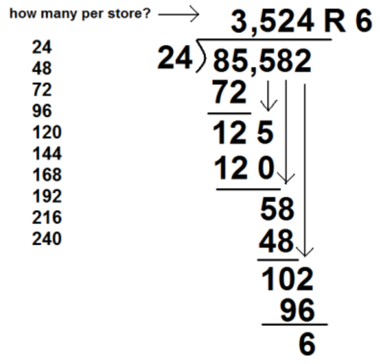

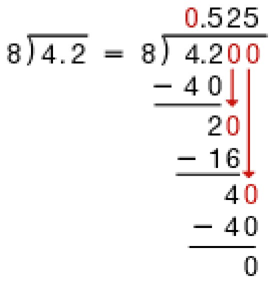

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Expressing the remainder

Children are required to be able to express the remainder in a number of different formats i.e. as whole numbers, fractions, decimals fractions; or rounded up or down. The format will depend upon the context of the question

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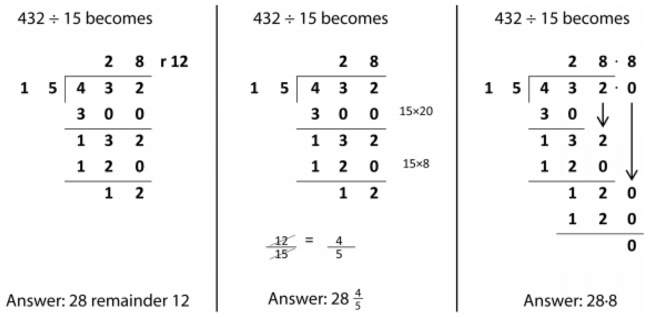

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